# Towards a Nonlinear Model Predictive Control for Quadrupedal Locomotion on Rough Terrain

Angelo Bratta[1], Niraj Rathod[1,2], Mario Zanon[2], Octavio Villarreal[1], Alberto Bemporad[2], Claudio Semini[1] and Michele Focchi[1]

***Abstract*— In this extended abstract, we give a short introduction to our ongoing work [1] on a real-time Nonlinear Model Predictive Control (NMPC) tailored to a legged robot for achieving dynamic locomotion on a wide variety of terrains. We introduce a mobility-based criterion to define an NMPC cost that enhances the locomotion of quadruped robots while maximizing leg mobility and staying far from kinematic limits. In addition, we include a cost term to regularize Ground Reaction Forces (GRFs) inside friction cone. We demonstrate the efficiency of our approach executing an omni-directional motion on our Hydraulically actuated Quadruped (HyQ) robot and showing in simulation a walk into a V-shaped chimney.**

## I. Introduction

The main advantage of legged robots with respect to their wheeled counterpart is their ability to traverse complex and unstructured environments such as forests, obstacles, and debris. An online re-planning is required since it can intrinsically cope with the problem of error accumulation in planned motion in such real scenarios. For online re-planning, MPC has gained broad interest in the robotics community for legged locomotion, since it considers information about the future states of the robot and hence can assure recursive feasibility. In our work [1], we choose a simplified model defined in an optimization-friendly way and the real-time iteration [2] scheme for the NMPC. We run our NMPC in real-time on the HyQ robot [3]. A careful design of the cost function of the NMPC is essential to achieve successful experiments. While tracking references for states and controls, we introduce a term that penalizes the distance between hip-to-foot and the reference value of maximum *mobility* (attitude of a leg to arbitrarily change foot position [4]). We use a simpler model and differently from [5] that re-plans at each touchdown, we are able to run the planner in an MPC fashion. In addition, we include a term that increases the force robustness. Especially in some scenarios (Fig. 1) it is desirable to keep the GRFs as close as possible to the center of the friction cone. A similar approach has been proposed by [6], [7], but integrating it in a NMPC allows to cope with uncertainties in the contact normal estimation and external disturbances.

## II. NMPC

Our planning scheme uses a real-time NMPC formulation which solves an Optimal Control Problem based on the current state $\hat{\mathbf{x}}_0$ of the system. We define the decision variables as the predicted state and control input given by $\mathbf{x}^{\mathrm{p}} = \{\mathbf{x}_0, \ldots, \mathbf{x}_N\}$ and $\mathbf{u}^{\mathrm{p}} = \{\mathbf{u}_0, \ldots, \mathbf{u}_{N-1}\}$, respectively, such that a Nonlinear Programming (NLP) formulation can be stated as:

$$\min_{\mathbf{x}^{\mathrm{p}},\mathbf{u}^{\mathrm{p}}} \sum_{k=0}^{N-1} \ell\left(\mathbf{x}_k, \mathbf{u}_k, \mathbf{a}_k\right) + \ell_{\mathrm{T}}\left(\mathbf{x}_N\right) \tag{1a}$$

$$\text{s.t.} \quad \mathbf{x}_0 = \hat{\mathbf{x}}_\mathbf{0}, \tag{1b}$$

$$\mathbf{x}_{k+1} = f\left(\mathbf{x}_k, \mathbf{u}_k, \mathbf{a}_k\right), \qquad k \in \mathbb{I}_0^{N-1}, \tag{1c}$$

$$h\left(\mathbf{x}_k, \mathbf{u}_k, \mathbf{a}_k\right) \leq 0, \qquad k \in \mathbb{I}_0^{N-1}, \tag{1d}$$

where, $\ell : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_a} \to \mathbb{R}$ is the stage cost function consisting of tracking cost of the state and control inputs $\ell_{\mathrm{t}}$, cost related to mobility $\ell_{\mathrm{m}}$, and cost of the regularization of the GRFs inside the cone $\ell_{\mathrm{r}}$; $\ell = \ell_{\mathrm{t}} + \ell_{\mathrm{m}} + \ell_{\mathrm{r}}$. The term $\ell_{\mathrm{T}} : \mathbb{R}^{n_x} \to \mathbb{R}$ is the terminal cost function. The vector $\mathbf{a}_k$ includes model parameters. The nonlinear robot dynamics defined with the Single Rigid Body Dynamics model are introduced by the equality constraints (1c). The predicted state $\mathbf{x}_k$ is the vector of robot's Center of Mass (CoM) position, CoM velocity, base orientation and angular velocity. The control inputs $\mathbf{u}_k$ are the GRFs $f$. Finally, the path constraints are included with (1d) which, for example, can be unilateral and friction cone constraints associated to GRFs (square pyramid approximation).

## III. Force Robustness

It is well-known that in order to avoid foot slippage the GRFs must be inside the friction cone. However, either the disturbances acting on the system or the wrong estimation of parameters (e.g., contact normals, friction coefficient) could result in moving the force outside of the cone, causing a loss of contact. Thus, robustness in the GRFs is required. In this work, we penalize GRFs that are in the vicinity of the cone boundaries, thanks to the term $\ell_{\mathrm{r}}$ in the cost.

$$\ell_{\mathrm{r}} = \| \, _{\mathcal{K}}\mathbf{u}_k \, \|_{\mathbf{P}}^2 \tag{2}$$

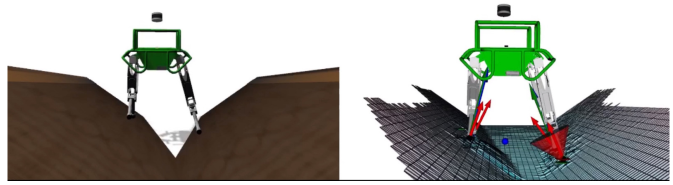

Fig. 1. Simulation of the HyQ robot walking inside a V-shaped chimney. Red cone corresponds to the friction cone, the arrows to the contact forces.

[1] The authors are with the Dynamic Legged Systems Lab, Istituto Italiano di Tecnologia (IIT), Genova, Italy. name.surname@iit.it

[2] The authors are with the IMT School for Advanced Studies Lucca, Lucca, Italy name.surname@imtlucca.it

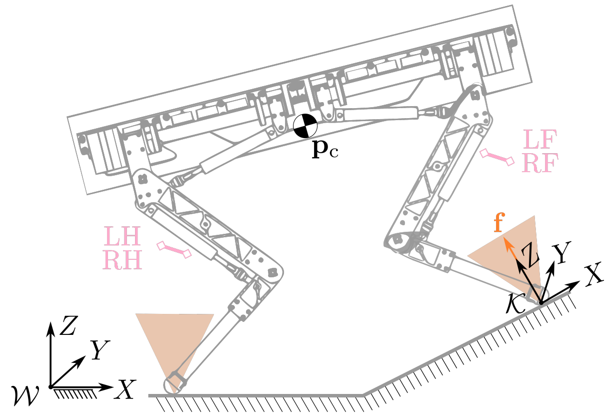


Fig. 2. HyQ schematic showing the inertial frame ($\mathcal{W}$), and the contact frame ($\mathcal{K}$). $\mathbf{p}_c$ corresponds to the position of the CoM.

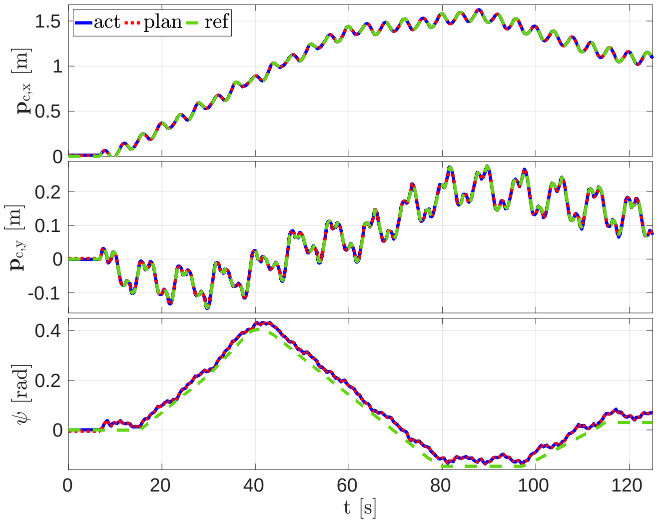


Fig. 3. Experiment: CoM X-Y position and yaw $\psi$ in omni-directional walk experiment. The blue, dotted red and dashed green line represent the actual, planned and reference values, respectively.

The term ${}_{\mathcal{K}}\mathbf{u}_k$ corresponds to the force, rotated in the *Contact frame* $\mathcal{K}$, see Fig. 2. Such frame is located in the contact point of the foot and has its Z axis aligned with the normal of the terrain, while X-Y axes are parallel to the ground. The weight matrix P is such that it penalizes the tangential components of GRFs in the contact frame $\mathcal{K}$ to obtain the resultant GRFs as close as possible to the contact normals.

## IV. Results

Our NMPC re-plans online at a frequency of 25 Hz with a prediction horizon of 2 seconds, sampling time 0.04 s (50 nodes). We tested our NMPC in several simulations and experimental scenarios [1] and here we briefly present some of them. The first experiment shows the omni-directional walk performed by HyQ with the NMPC on a flat terrain. In this experiment, the robot was commanded with a longitudinal velocity ${}_{\mathcal{H}}\mathbf{v}_{\mathrm{c,x}}^{\mathrm{usr}}$ given by the user to walk forward/backward and then at a lateral velocity. Finally, a heading velocity $\omega_{\mathrm{z}}^{\mathrm{usr}}$ was commanded to turn in the left/right direction. Figure 3 shows the CoM $X$-$Y$ position and yaw angle of the robot base and it can be noticed that the actual values track very closely the planned trajectories provided by the NMPC. In addition, we exploited the cost term related to force robustness to perform a simulation in which HyQ walks at $0.03\,\mathrm{m/s}$ commanded velocity in the $X$ direction into a V-shaped chimney with friction coefficient $\mu = 0.7$ and walls inclined at $35°$ to the ground, see Fig.1. Contact normals obtained through an RGB-D camera are used to formulate the force robustness cost (2) in the contact frame $\mathcal{K}$. Fig. 4 shows that the longitudinal and lateral components of the GRF at the Left-Front (LF) foot stay within the bound $\mu f_{\mathrm{z}}$ (in red) imposed by the cone constraints.

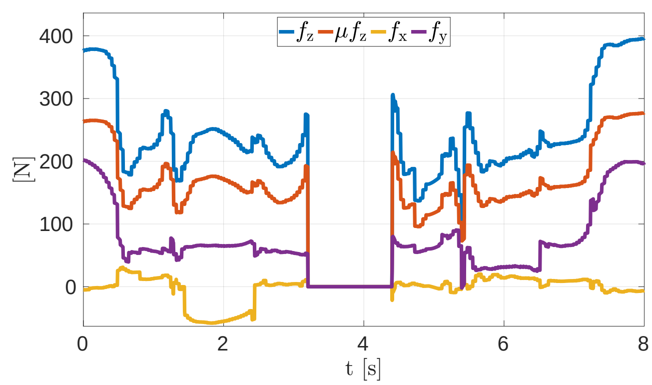


Fig. 4. Simulation: walk into a V-shaped chimney. GRFs of LF leg for a single gait cycle with cone constraints and regularization cost. Both the longitudinal $f_{\mathrm{x}}$ and lateral $f_{\mathrm{y}}$ lie conservatively within the bound $\mu f_{\mathrm{z}}$ imposed by cone constraints.

## V. Conclusions

In this extended abstract of our ongoing work [1], we present an NMPC scheme that allows us to perform successful experiments of omni-directional motions. In addition, thanks to a force robustness term in the cost term, the NMPC provides feasible trajectories also in a $35°$ V-shaped chimney. Future works will include disturbance rejection feature and foothold optimization.

## References


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