

## Actuator Sizing for Highly-Dynamic Quadruped Robots Based on Squat Jumps and Running Trots

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It is challenging to design a quadruped robot that can perform highly dynamic tasks like jumping and running. Estimating appropriate range of joint torques and velocities is essential for the selection of the leg actuators. Jumping and running are considered as extreme tasks that push the actuators to their limits. In this paper we proposed a simple method that allows a quadruped robot designer to obtain approximate peak joint torques and joint velocities needed for a running trot at various forward velocities and squat jumps at different heights. A SLIP model is used for the mapping of CoM trajectory of a quadruped robot during a running trot. Experiments for a squat jump and running trot are performed with the highly dynamic quadruped robot HyQ for the validation of the proposed approaches. A case study is also discussed to demonstrate the usage of proposed tool.

*Keywords:* Quadruped Robot Design; Scaling; Actuator Sizing; Running Trot.

### 1. Introduction

Legged robots have become a much more prominent field of robotics in recent years, due to their potential versatility and capability of performing tasks that conventional vehicles are unable to do. Most of the recent legged robots, however, lack the versatility of performing both precise navigation over rough terrain and the strong, fast motions that are necessary for dynamic tasks such as jumping and running. Presently very few examples

exist besides BigDog<sup>1</sup> by Boston Dynamics and HyQ<sup>2</sup> by IIT.

To design such kind of machines, firstly the designer has to define the tasks that the robot should accomplish then choose the actuator that satisfies the task requirements in terms of joint velocity and torque. It is easier for a designer to narrow down the desired dimensions and mass of the robot. But when it comes to the selection of actuators, it is not trivial to obtain appropriate joint velocity and torque limits without correctly modeling the whole robot in simulation and implementing stable locomotion controllers. The aim of this work is to build a scaling tool that helps the quadruped robot designers to select actuators that fulfill the desired requirements. We started our study using the squat jump as a characteristic motion for highly dynamic robots, obtaining peak values of joint velocity and torque in relation to robot mass, leg segment length and desired jump height.<sup>3</sup> This paper presents the extension of that study with a running trot to get peak torques and velocities in relation to forward velocity, robot mass and leg segment length. The validation of the proposed methodology is confirmed by performing a squat jump and running trot experiments with our highly dynamic quadruped robot HyQ.

## 2. State of the Art

In the early 1980's in the field of applied zoology, a number of experiments were conducted to study the dynamic similarities in quadruped mammals.<sup>4-6</sup> The hypotheses formulated by these studies layout the preliminary bio-inspired sizing guideline for the legged robot designers. Alexander<sup>5</sup> found that the legged animals of different sizes tend to move in dynamically similar fashion whenever their Froude numbers<sup>a</sup> are equal. In the light of dynamic similarity concept, it is also predicted that the geometrically similar animals of different sizes exhibit equal duty factors and equal relative stride lengths ( $stridelength/h$ ), when they are traveling with equal Froude number.<sup>6,8</sup> To use these biologically inspired criteria for a legged robot design, a quadruped robot designer needs to solve an optimization problem by taking care of desired goals of machine.

Similar studies were conducted for the electric DC motor sizing of a bounding robot<sup>9</sup> and a hopping monopod robot.<sup>10</sup> But these studies were limited to quadruped robots with telescopic legs or a specific type of actuator. Another scaling study is done for Oncilla-robot (slightly bigger ver-

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<sup>a</sup>Froude number is a dimensionless ratio used to study trends in animal gait patterns where the gravitational forces are important<sup>7</sup> and it is defined as  $\frac{v^2}{gh}$ , where  $v$  is forward velocity,  $g$  is the acceleration of free fall and  $h$  is the height of hip joint from the ground.

sion of successor CheetahCub-robot)<sup>11</sup> but the study is only limited to pantograph-based leg quadruped robots. The current work is for quadruped robots with articulated legs and it is not restricted to only one type of actuators.

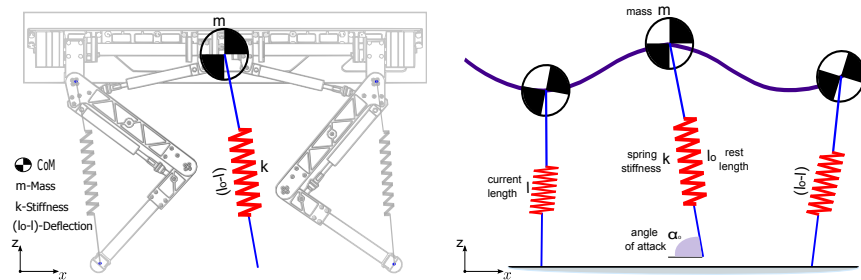


Fig. 1. (a) A quadruped robot with the equivalent virtual spring (connected to CoM) represents the virtual linear spring of a diagonal pair of legs (b) A CoM trajectory (purple solid) of the legged robot during a running trot represented by the SLIP model.

### 3. Simulation

**Running Trot Simulation** Desired joint torques are generated by placing a virtual linear spring between the hip and foot of each articulated robot leg<sup>12</sup> to control the motion of the leg during running. Each virtual spring of a diagonal leg pair is working synchronously in a trot and can be represented by one equivalent virtual spring, Fig. 1a. Blickhan and Full showed in one of their studies that the running motion for multilegged locomotion behaves like a bouncing monopod and they calculated the compression of a virtual monopod's leg and its stiffness from the animals' mechanical-energy fluctuation and ground-reaction force.<sup>13</sup> The center of mass (CoM) motion of any quadruped during a running trot can be approximated by a spring-loaded inverted pendulum (SLIP) model as it is observed in animals,<sup>14</sup> Fig. 1b. A point mass  $m$  attached to a mass-less linear spring can be described as a SLIP model where  $k$  is the spring stiffness,  $l$  is the current length and  $l_0$  is the rest length. During stance phase, the spring force exerted on the ground is defined as  $k(l_0 - l)$ . During flight phase, the mass follows a parabolic trajectory under the law of gravitation. We define the spring rest length in relation to the leg segment length  $l_{ls}$  (Fig. 2b) of each articulated robot leg as follows:  $l_0 = K_l l_{ls}$ , where  $K_l$  is the ratio between the leg segment length and the spring rest length. In our simulation  $K_l$  is equal to 1.41, which we assume to be a good value based on our experience with HyQ. From the

SLIP model, we know the foot trajectory in Cartesian coordinates of the equivalent virtual spring leg during stance. Using the leg Jacobian, we can calculate the angular position, velocity and acceleration of hip and knee joints of the articulated robot leg. During the stance phase of a quadruped running trot only two feet touch the ground. So the ground reaction forces (GRF) for each articulated robot leg can be defined as  $F = F_{vl}/2$  assuming evenly distributed load, where  $F_{vl}$  is the force for the equivalent virtual spring. The leg's GRF is then transformed into hip and knee joint torques with the Jacobian transpose.

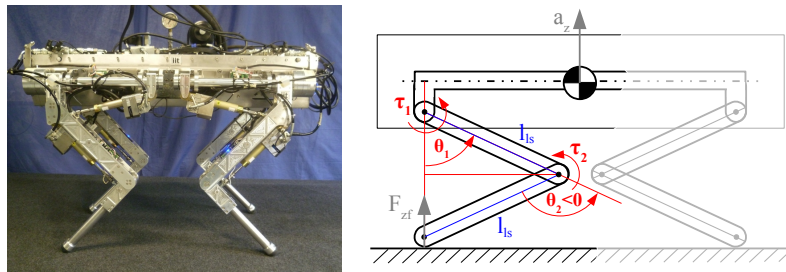


Fig. 2. HyQ: Hydraulic Quadruped robot. (a) picture and (b) sketch of side view of robot in squat posture, defining the centre of mass (CoM); acceleration vector  $a_z$ ; joint angles  $\theta_{1,2}$  and torques  $\tau_{1,2}$  of the hip and knee joint, respectively; leg segment lengths  $l_{1s}$  and ground reaction force vector at the foot  $F_{zf}$ .

**Squat jump Simulation** As we published in<sup>3</sup> a squat jump is composed of several phases: first, a vertical *acceleration phase* from a squatting posture until lift-off; then, a parabolic *flight phase* with the legs moving to a suitable landing posture. The jump height is the crucial input to our simulation and is measured as the vertical distance that the CoM travels from the time the body lifts off the ground to the end of the upward motion. The maximum jump height of any object undergoing a parabolic flight phase is directly related to its lift-off velocity  $v_{lo}$  which can be obtained by equating the kinetic energy at lift-off with the potential energy at the maximum jump height. The estimated  $v_{lo}$  leads to the required constant vertical acceleration  $a_z$  and vertical force  $F_{zf}$  during the acceleration phase. The maximum vertical force is transformed into maximum knee joint torque with the Jacobian transpose. The above mentioned parameters are illustrated in Fig. 2b.

#### 4. Experimental Platform: HyQ robot

The experimental platform used in this study is the versatile, quadruped robot HyQ,<sup>15</sup> Fig. 2a, a hydraulically and electrically actuated machine

that weighs  $70kg$ , is  $1m$  long and has a leg segment length of  $0.35m$ . The robot's legs have three degrees of freedom each, two hydraulic joints in the sagittal plane (hip and knee flexion/extension) and one electric joint for hip abduction/adduction. Each joint has  $120^\circ$  range of motion and is controllable in torque and position. The maximum joint torque is  $145Nm$  for the hydraulic and  $152Nm$  for the electric joints.

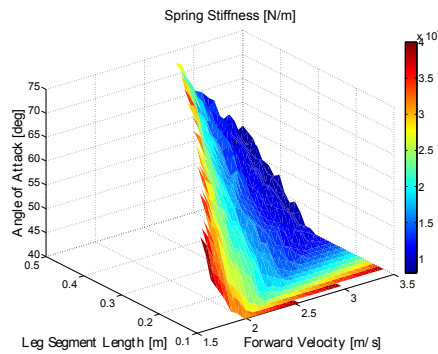


Fig. 3. Plot of stable SLIP parameters(Spring Stiffness and Angle of Attack) for different leg segment lengths and forward velocities at robot mass  $73kg$ .

## 5. Results

Using the SLIP model we performed a number of trotting simulations for a range of forward velocities and leg segment lengths. First of all, suitable SLIP parameters (spring stiffness and angle of attack) had to be calculated for each input pair based on the steps-to-fall map.<sup>16</sup> The SLIP parameters with the lowest spring stiffness that resulted in stable hopping of 50 or more steps were then selected, shown in a Fig. 3. This way low impact peaks were obtained. A running trot experiment is performed on HyQ at forward velocity  $1.6\text{ m/s}$  with 40% duty factor and it gives peak hip and knee joint torques equal to  $24.3\text{ N/m}$  and  $115.5\text{ N/m}$  respectively. 'HyQ Exp' arrows shown in Fig. 4 are the estimated peak hip and knee joint torques for a running trot at  $1.6\text{ m/s}$ , equal to  $21.9\text{ Nm}$  ( $0.3Nm/kg * 73kg$ ) and  $116.8\text{ Nm}$  ( $1.6Nm/kg * 73kg$ ) respectively. Fig. 4 shows the peak joint torque scaled by the robot's body weight (BW) for different leg segment lengths and forward velocities. White areas indicate where the SLIP model failed to perform 50 steps for the given parameter range.

For a squat jump, Fig. 5a is a validation of proposed approach by comparing HyQ's experimental data with simulation.<sup>3</sup> The peak torques scaled by the BW are shown in Fig. 5b for different leg segment lengths and jump

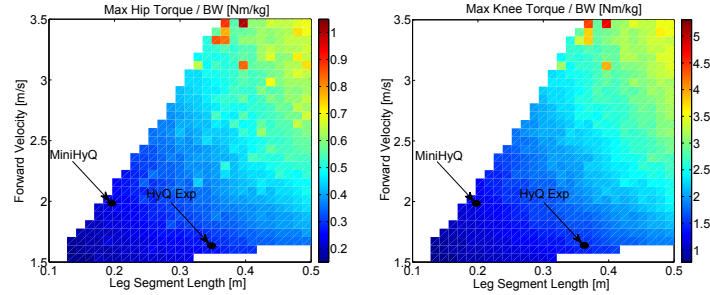


Fig. 4. (a) Plot of maximum hip joint torque scaled by BW for different leg segment lengths and forward velocities. (b) Plot of maximum knee joint torque. The arrows show the estimated result according to desired task specification for MiniHyQ (see Section 6).

heights. The three subplots of Fig. 5a show the data of the experiment (red solid line) and of the simulation (black dashed line) for the knee joint angle (top), knee joint torque (middle) and vertical ground reaction force (bottom). The acceleration phase starts at 0.1s and lasts till 0.4s when the torques go to zero. The robot touches down again at 0.78s. The simulation calculates values only during the acceleration phase.

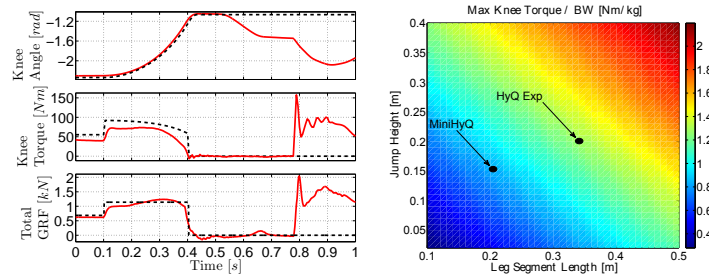


Fig. 5. (a) Plot of experimental data (red solid) and simulated data (black dashed) for a squat jump motion of 0.2m jump height. *Top*: knee joint angle  $\theta_2$ ; *middle*: knee torque  $\tau_2$ ; *bottom*: total ground reaction force  $4F_{zf}$ . (b) plot of maximum knee joint torque scaled by the BW for different leg segment lengths and jump heights. The arrows show HyQ experimental and MiniHyQ estimation results.

## 6. A case Study: MiniHyQ

A case study is done for the development of a smaller version of a highly dynamic and versatile quadruped robot named *MiniHyQ*. MiniHyQ will be a torque-controlled versatile robot of small size that will be able to walk, move over rough terrain, jump and run. *Desired specifications for MiniHyQ*: robot mass 15 kg and leg segment length is 0.2 m. *Task performance limits*:

capable of running at 2 m/s and jump 0.15 m. *Estimated peak torques:* for a *running trot* peak hip and knee joint torques are equal to 5.25 Nm ( $0.35Nm/kg * 15kg$ ) and 22.5 Nm ( $1.5Nm/kg * 15kg$ ) respectively. Fig. 5b shows for a *squat jump* an estimated maximum knee joint torque of 13.5 Nm ( $0.9Nm/kg * 15kg$ ) for MiniHyQ.

## 7. Discussion

In running trot simulation, it is assumed that the robot mass is evenly distributed and that the robot torso is always parallel to the ground. This assumption does not significantly influence the results even the biological studies have shown that all legged animals typically run with similar center of mass (CoM) motions relative to the (approximately) horizontal ground.<sup>17</sup> In this work ground stiffness is assumed constant and the equivalent virtual spring stiffness and angle of attack is chosen from the stability domain of steps-to-fall map of the SLIP model. Assumptions and limitation are discussed in detail for a squat jump simulation.<sup>3</sup>

## 8. Conclusion and Future Works

Above mentioned case study is an exemplary display of proposed tool for designing a versatile quadruped robot. This paper presented a study of actuator sizing for highly-dynamic quadruped robots and showed how one can estimate torque limits for tasks like a squat jump and running trot at various jump heights and forward velocities, respectively.

**Future Works** This study will be extended for the estimation of the hip abduction/adduction joint peak torques. Currently we are building a MiniHyQ (a smaller version of HyQ), whose actuators are sized and selected based on these estimations. These simple presented methods will also be validated by performing experiments on the MiniHyQ.

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